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Solution by the PROPOSER.

From $b=ai\tan\frac{m\pi}{n}$, where $i=\sqrt{-1}$, we derive $\frac{a+b}{a-b}=\frac{1+i\tan\frac{m\pi}{n}}{1-i\tan\frac{m\pi}{n}}$

$$=\frac{1-\tan^2\frac{m\pi}{n}+2i\tan\frac{m\pi}{n}}{1+\tan^2\frac{m\pi}{n}}=\cos^2\frac{m\pi}{n}-\sin^2\frac{m\pi}{n}+2i\sin\frac{m\pi}{n}.\cos\frac{m\pi}{n}=\cos\frac{2m\pi}{n}$$

$+i\sin\frac{2m\pi}{n}$, the n^{th} power of which, by De Moivre's Theorem, equals 1.

$\therefore (a+b)^n=(a-b)^n$.

Also solved by *F. P. MATZ.*

51. Proposed by J. W. NICHOLSON, LL. D., President, and Professor of Mathematics, Louisiana State University and A. and M. College, Baton Rouge, Louisiana.

Solve the equation $x^5+5mx^3+5m^2x+n=0$.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let $x=y+z$. The equation then easily reduces to $y^5+z^5+5(yz+m)\{y^3+z^3+(2yz+m)(y+z)\}+n=0$. Now x may be decomposed into two parts, y and z, in an infinite variety of ways, and we may, therefore, suppose y and z are such as to satisfy the condition $yz+m=0$. This gives $yz=-m$, $y^5+z^5=-n$. Let $y^5=t_1$, $z^5=t_2$, then we have $t_1t_2=-m^5$, $t_1+t_1=-n$. $\therefore t_1$ and t_2 are the roots of the equation $t^2+nt-m^5=0\ldots\ldots(1)$. $\therefore x=y+z=\sqrt[5]{t_1}+\sqrt[5]{t_2}$.

Case I. When m is positive.

Let $t=u\sqrt{m^5}$, then (1) becomes $u^2+\frac{n}{\sqrt{m^5}}u-1=0$; but $\tan^2\frac{1}{2}\theta+2\cot\theta\tan\frac{1}{2}\theta-1=0$. $\therefore 2\cot\theta=n/\sqrt{m^5}$, or $\tan\theta=2\sqrt{m^5}/n$. $\therefore t_1=\sqrt{m^5}\tan\frac{1}{2}\theta$, $t_2=-\sqrt{m^5}\cot\frac{1}{2}\theta$. $\therefore x=\sqrt{m}(\sqrt[5]{\tan\frac{1}{2}\theta}-\sqrt[5]{\cot\frac{1}{2}\theta})$, where $\theta<90°$. Four of the five roots of $\sqrt[5]{\tan\frac{1}{2}\theta}$ are imaginary. Let $\tan\frac{1}{2}\phi=r=$the real value of $\sqrt[5]{\tan\frac{1}{2}\theta}$, and let a_1, a_2, a_3, a_4 represent the four imaginary roots of unity

$\frac{\sqrt{5}-1+\sqrt{-10-2\sqrt{5}}}{4}$, $\frac{\sqrt{5}-1-\sqrt{-10-2\sqrt{5}}}{4}$, $\frac{-\sqrt{5}-1-\sqrt{-10+2\sqrt{5}}}{4}$, $\frac{-\sqrt{5}-1+\sqrt{-10+2\sqrt{5}}}{4}$, respectively.

Then $x_1=\sqrt{m}(r-\frac{1}{r})$, $x_2=\sqrt{m}(ra_1-\frac{1}{ra_1})$, $x_3=\sqrt{m}(ra_2-\frac{1}{ra_2})$, $x_4=\sqrt{m}(ra_3=\frac{1}{ra_3})$, $x_5=\sqrt{m}(ra_4-\frac{1}{ra_4})$. Substituting the values of r, a_1, a_2, a_3, a_4, we get $x_1=-2\sqrt{m}\cot\phi$,

$$x_2=-\tfrac{1}{2}\sqrt{m}\{(\sqrt{5}-1)\cot\phi-\sqrt{-10-2\sqrt{5}}\,\text{cosec}\phi\},$$
$$x_3=-\tfrac{1}{2}\sqrt{m}\{(\sqrt{5}-1)\cot\phi+\sqrt{-10-2\sqrt{5}}\,\text{cosec}\phi\},$$
$$x_4=\tfrac{1}{2}\sqrt{m}\{(\sqrt{5}+1)\cot\phi+\sqrt{-10+2\sqrt{5}}\,\text{cosec}\phi\},$$
$$x_5=\tfrac{1}{2}\sqrt{m}\{(\sqrt{5}+1)\cot\phi-\sqrt{-10+2\sqrt{5}}\,\text{cosec}\phi\}.$$

Case II. When m is negative and $-4m^5<n^2$.

Then (1) becomes $t^2+nt+(-m^5)=0$, or $u^2+\dfrac{n}{\sqrt{-m_5}}u+1=0$; but $\tan^2\tfrac{1}{2}\theta-2\text{cosec}\theta\tan\tfrac{1}{2}\theta+1=0$. $\therefore -2\text{cosec}\theta=n/\sqrt{-m^5}$, or $\sin\theta=-2\sqrt{-m^5}/n$.

$\therefore t_1=\sqrt{-m^5}\tan\tfrac{1}{2}\theta$, $t_2=\sqrt{-m^5}\cot\tfrac{1}{2}\theta$. $\therefore x=\sqrt{-m}(\sqrt[5]{\tan\tfrac{1}{2}\theta}+\sqrt[5]{\cot\tfrac{1}{2}\theta})$.

By a process similar to that in Case I, we get,

$$x_1=\sqrt{-m}(r+\tfrac{1}{r})=2\sqrt{-m}\,\text{cosec}\phi,$$
$$x_2=\sqrt{-m}(ra_1+\tfrac{1}{ra_1})=\tfrac{1}{2}\sqrt{-m}\{(\sqrt{5}-1)\text{cosec}\phi-\sqrt{-10-2\sqrt{5}}\cot\phi\},$$
$$x_3=\sqrt{-m}(ra_2+\tfrac{1}{ra_2})=\tfrac{1}{2}\sqrt{-m}\{(\sqrt{5}-1)\text{cosec}\phi+\sqrt{-10-2\sqrt{5}}\cot\phi\},$$
$$x_4=\sqrt{-m}(ra_3+\tfrac{1}{ra_3})=-\tfrac{1}{2}\sqrt{-m}\{(\sqrt{5}+1)\text{cosec}\phi-\sqrt{-10+2\sqrt{5}}\cot\phi\},$$
$$x_5=\sqrt{-m}(ra_4+\tfrac{1}{ra_4})=-\tfrac{1}{2}\sqrt{-m}\{(\sqrt{5}+1)\text{cosec}\phi+\sqrt{-10+2\sqrt{5}}\cot\phi\}.$$

Case III. When m is negative and $-4m^5<n^2$.

In this case the preceding method fails. Let $x=ku$, then $x^5+5mx^3+5m^2x+n=0$ becomes $u^5+\dfrac{5mu^3}{k^2}+\dfrac{5m^2u}{k^4}+\dfrac{n}{k^5}=0$; also $\cos^5\theta-\tfrac{5}{4}\cos^3\theta+\tfrac{5}{16}\cos\theta-\tfrac{1}{16}\cos5\theta=0$. Let $u=\cos\theta$, then $5m/k^2=-5/4$,

$$\therefore k=2\sqrt{-m},\ -\tfrac{1}{16}\cos5\theta=\frac{n}{k^5}=\frac{n}{32m^2\sqrt{-m}},\quad \therefore \cos5\theta=-\frac{n}{2m^2\sqrt{-m}}.$$

Since $-m$ is positive, this gives five real values of θ, to be taken $<90°$.

$$\therefore x_1=2\sqrt{-m}\cos\theta,\ x_2=2\sqrt{-m}\cos(\frac{2\pi}{5}-\theta),\ x_3=2\sqrt{-m}\cos(\frac{2\pi}{5}+\theta),$$
$$x_4=2\sqrt{-m}\cos(\frac{4\pi}{5}-\theta),\ x_5=2\sqrt{-m}\cos(\frac{4\pi}{5}+\theta).$$

II. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

Let $x=z-\dfrac{m}{z}$, and the equation reduces to $z^{10}+nz^5=m^5$.

Whence $z=\sqrt[5]{-\dfrac{n}{2}\pm\sqrt{4m^5-n^2}}$.

III. Solution by H. C. WILKES, Skull Run, West Virginia; and A. H. BELL, Hillsborough, Illinois.

Factoring, etc., $(x^2+2m)(x^2+3m)=m^2-\frac{n}{x}$. Let $x=n$; then $x=n$, $\pm\sqrt{-m-1}$, $\pm\sqrt{1-2m}$, which will be the five roots.

Or $(x^2+3m)(x^2+2m)=(m+\sqrt{\frac{n}{x}})(m-\sqrt{\frac{n}{x}}$. Assuming $x^3+3m=\sqrt{\frac{n}{x}}$, $x^3+2m=m-\sqrt{\frac{n}{x}}$. $x=\sqrt{-\frac{3m}{2}}$, $m=2\sqrt{\frac{n}{x}}$; hence $x=\frac{4n}{m^2}$. Substituting $m=2\sqrt{\frac{n}{x}}$ for m in eq. 1, $x^5+10x^3\sqrt{\frac{n}{x}}+21n=0$. This can be developed, $x^{10}-58nx^5+441=0$. $\therefore x=\sqrt[5]{49n}$ or $\sqrt[5]{9n}$.

[The above is not strictly a solution, but affords a method of discovering integer roots, if any. The solu-of Professor Zerr is especially full and neat. EDITOR.]

Also solved by *F. P. MATZ.*

52. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

In how many ways can we arrange 12 friends of the MONTHLY, around a table, so that; (1) the editors may never be together, (2) Matz and Halsted may never be apart, and (2) Zerr and Ellwood may always have Gruber betwixt them?

Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

I. Considering one editor in position the other may occupy 9 places; but the first editor may take 12 places, and therefore the two take 108 positions. For each of these places the remaining nine mathematicians may be seated in $\underline{|9}$ ways, making $108\underline{|9}$ ways altogether.

II. If Matz and Halsted are never apart we may consider them as an element to be arranged as *each* of the other individuals. We then have $\underline{|11}$ ways of arranging them without regarding the *internal* arrangement of the group; this may be arranged in two ways. We, therefore, have $2\underline{|11}$ as the number of arrangements.

III. By the same reasoning as in the last case we have the number of arrangements $=2\underline{|10}$.

NOTE.—No solution of problem 53 has as yet been received. The published solution of problem 49, in last issue, should have been credited to Prof. J. H. Grove, Howard Payne College, Brownwood, Texas.

PROBLEMS.

59. Proposed by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Demonstrate the identity $2^{2n+1}\frac{d^n}{dx^n}\left(x^{n+\frac{1}{2}}\frac{d^{n+1}}{dx^{n+1}}e^{\sqrt{x}}\right)=e^{\sqrt{x}}$.